Feedback Enhances Simultaneous Energy and Information Transmission in Multiple Access Channels

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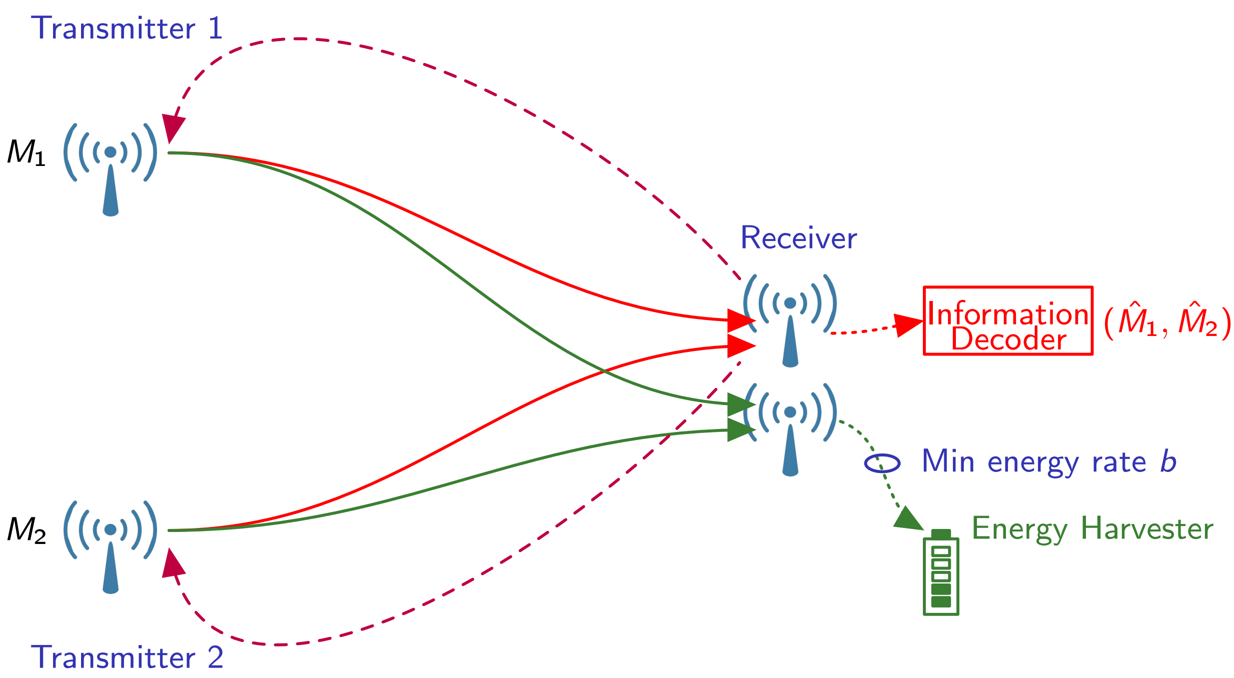
Multi-Access Channel With Feedback and Minimum Energy Rate Constraint b



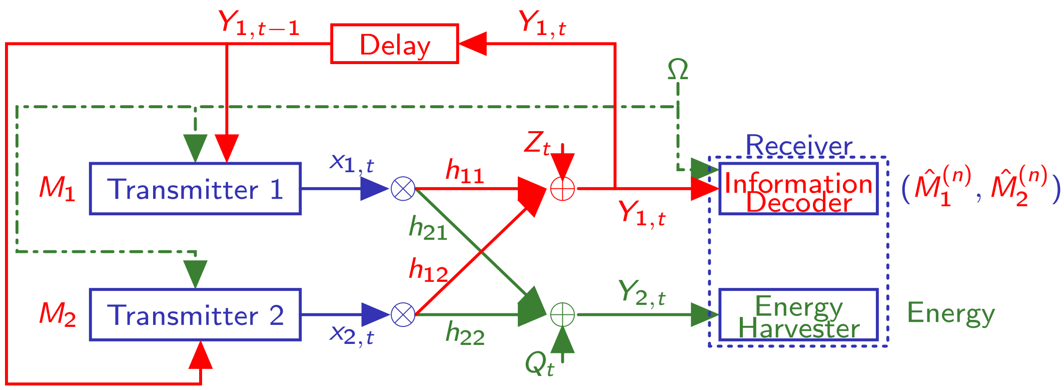
Trade-off Between Information and Energy Transmission

State-of-the-Art

- Point-to-point channels:
  - Basic p-2-p communication link [Varshney'2008]
  - Parallel p-2-p channel (frequency-selective) [Grover and Sahai'2010]

- More complex network structures:
  - Multi-hop networks [Gurakan *et al.*'2012], [Fouladgar and Simeone'2012]
  - Multiple-access channels [Gastpar'2004], [Fouladgar and Simeone'2012]
  - Broadcast channels [Benawan and Ulukus'2016], [Zhang and Ho'2013], [Xiang and Tao'2012]
  - Wiretap channels [Benawan and Ulukus'2014]
  - Interference channels [Park and Clerckx'2013]
  - Interactive two-way channels [Popovski et al.'2013]
  - Unicast and multicast networks [Fouladgar and Simeone'2013]

Gaussian Multiple Access Channel With Feedback and Energy Harvester



Information Decoder: $Y_{1,t} = h_{11}X_{1,t} + h_{12}X_{2,t} + Z_t$

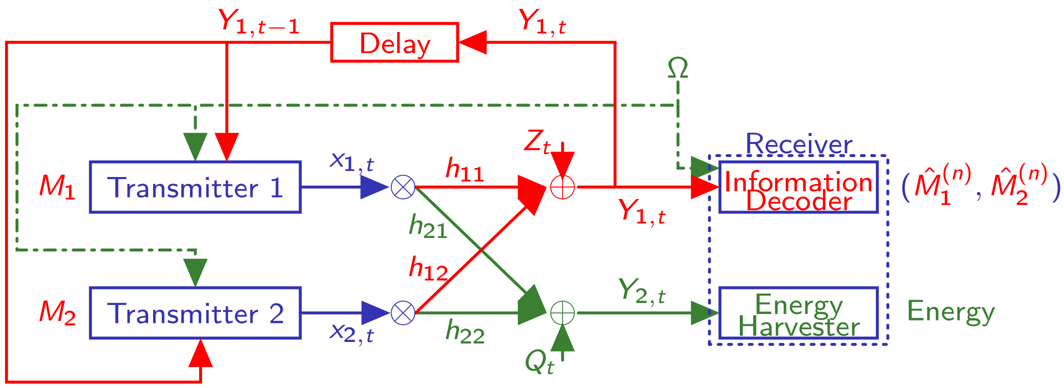
Energy harvester: $Y_{2,t} = h_{21}X_{1,t} + h_{22}X_{2,t} + Q_t$

- n: blocklength; $X_{1,t}, X_{2,t}, Q_t, Z_t \in \mathbb{R}$; $\{Z_t\}, \{Q_t\}$ i.i.d. $\sim \mathcal{N}(0,1)$
- Constant $h_{11}, h_{12}, h_{21}, h_{22} \geqslant 0$ with:

$$\forall j \in \{1,2\}, \|\mathbf{h}_j\|_2 \leq 1, \text{ with } \mathbf{h}_j \triangleq (h_{j1}, h_{j2})^{\mathsf{T}} \quad \text{(energy conservation principle)}$$

- Input power constraints: $\frac{1}{n}\sum_{t=1}^{n} \mathrm{E}\left[X_{i,t}^2\right] \leqslant P_i, \quad i \in \{1,2\}$
- Fully described by signal-to-noise ratios: $\mathrm{SNR}_{ji} \triangleq |h_{ji}|^2 P_i, \quad (i,j) \in \{1,2\}^2$
- Perfect feedback: $X_{i,t} = f_{i,t}^{(n)}(M_i, Y_{1,1}, \ldots, Y_{1,t-1})$

Information Transmission

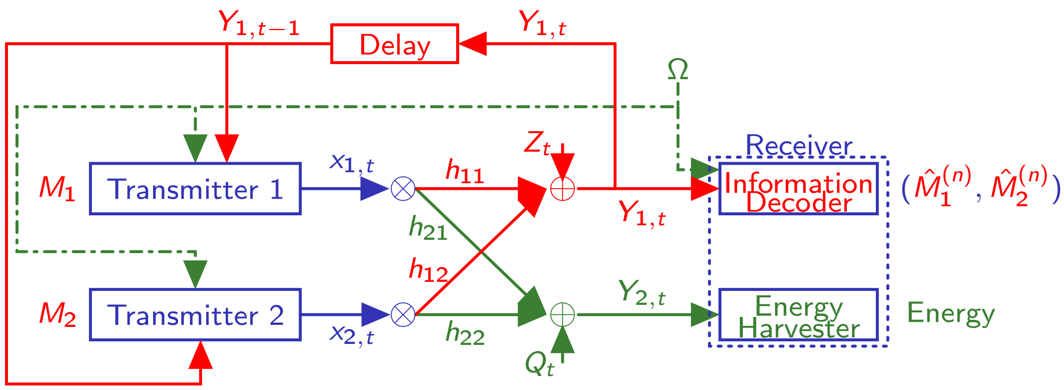


- Transmitters 1 and 2 send M_1 and M_2 to the information decoder
- Messages M_1 and M_2 independent ; $M_i \sim \mathcal{U}\{1, \ldots, 2^{nR_i}\}$
- R_1 and R_2 are information transmission rates
- Common randomness Ω known to all terminals

Probability of Error

$$P_{\text{error}}^{(n)}(R_1, R_2) \triangleq \Pr\left\{(\hat{M}_1^{(n)}, \hat{M}_2^{(n)}) \neq (M_1, M_2)\right\}$$

Energy Transmission



- Minimum energy rate b at EH (in energy units per channel use) such that

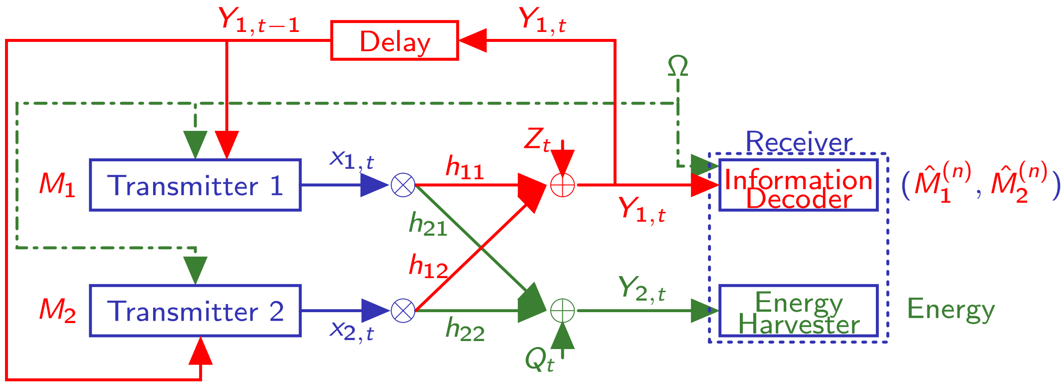
$$0 \leqslant b \leqslant 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{\text{SNR}_{21}\text{SNR}_{22}}$$

- Average energy rate: $B^{(n)} \triangleq \frac{1}{n}\sum_{t=1}^{n} Y_{2,t}^2$
- B energy rate such that with $b \leqslant B \leqslant 1 + \text{SNR}_{21} + \text{SNR}_{22} + 2\sqrt{\text{SNR}_{21}\text{SNR}_{22}}$
- Guarantee $B^{(n)} \geqslant B$ with high probability

Probability of Energy Outage

$$P_{\text{outage}}^{(n)}(B) \triangleq \Pr\left\{B^{(n)} < B - \epsilon\right\}, \ \epsilon > 0 \text{ arbitrarily small}$$

Simultaneous Energy and Information Transmission (SEIT)



Objective of SEIT

Provide blocklength-n coding schemes such that:

(i) information transmission occurs at rates R_1 and R_2 with $P^{(n)}_{\text{error}}(R_1, R_2) \to 0$; and

(ii) energy transmission occurs at rate B with $P^{(n)}_{\text{outage}}(B) \to 0$ and $B \geq b$.

Under these conditions, the information-energy rate-triplet (R_1, R_2, B) is achievable in the G-MAC-F with minimum energy rate b.

$\Rightarrow$ What are the fundamental limits on achievable information-energy rate-triplets?

Main Results: Perfect Feedback Information-Energy Capacity Region $\mathcal{E}_b^{\mathrm{FB}}(\mathrm{SNR}_{11}, \mathrm{SNR}_{12}, \mathrm{SNR}_{21}, \mathrm{SNR}_{22})$

Theorem: Perfect Feedback Information-Energy Capacity Region $\mathcal{E}_b^{\mathrm{FB}}(\mathrm{SNR}_{11}, \mathrm{SNR}_{12}, \mathrm{SNR}_{21}, \mathrm{SNR}_{22})$

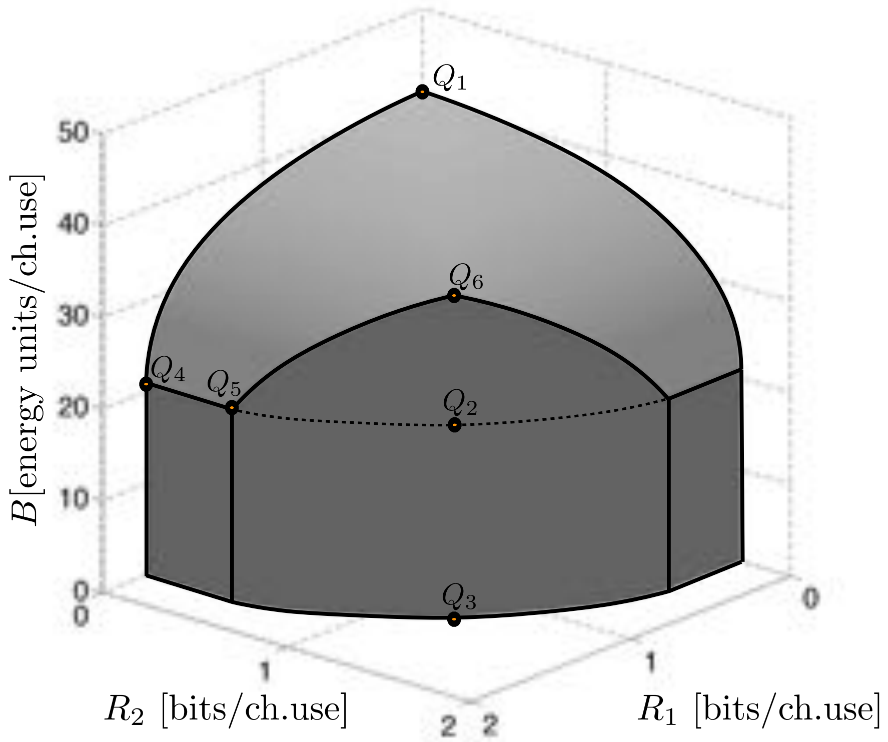
$\mathcal{E}_b^{\mathrm{FB}}(\mathrm{SNR}_{11}, \mathrm{SNR}_{12}, \mathrm{SNR}_{21}, \mathrm{SNR}_{22})$ contains all (R_1, R_2, B) that satisfy

$$
\begin{aligned}
0 \leqslant \quad R_1 \quad &\leqslant \frac{1}{2}\log_2\left(1 + \beta_1\, \mathrm{SNR}_{11}\left(1-\rho^2\right)\right),\\
0 \leqslant \quad R_2 \quad &\leqslant \frac{1}{2}\log_2\left(1 + \beta_2\, \mathrm{SNR}_{12}\left(1-\rho^2\right)\right),\\
0 \leqslant R_1 + R_2 &\leqslant \frac{1}{2}\log_2\left(1 + \beta_1\, \mathrm{SNR}_{11} + \beta_2\, \mathrm{SNR}_{12} + 2\rho\sqrt{\beta_1 \mathrm{SNR}_{11} \beta_2 \mathrm{SNR}_{12}}\right),\\
b \leqslant \quad B \quad &\leqslant 1 + \mathrm{SNR}_{21} + \mathrm{SNR}_{22} + 2\rho\sqrt{\beta_1 \mathrm{SNR}_{21}\ \beta_2 \mathrm{SNR}_{22}}\\
&\quad + 2\sqrt{(1-\beta_1)\mathrm{SNR}_{21}\ (1-\beta_2)\mathrm{SNR}_{22}},
\end{aligned}
$$

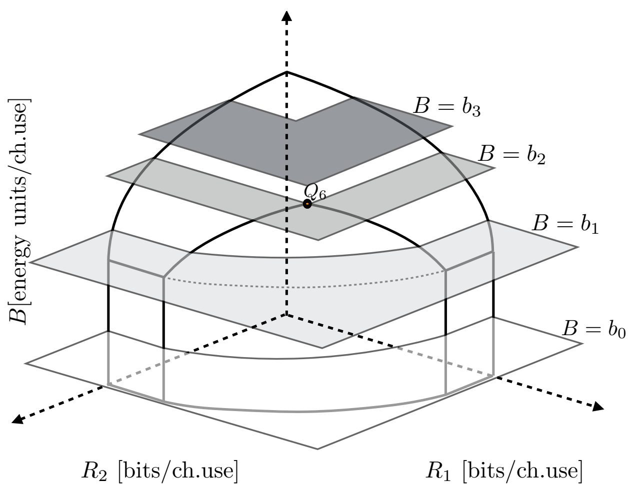
with $(\rho, \beta_1, \beta_2) \in [0,1]^3$.

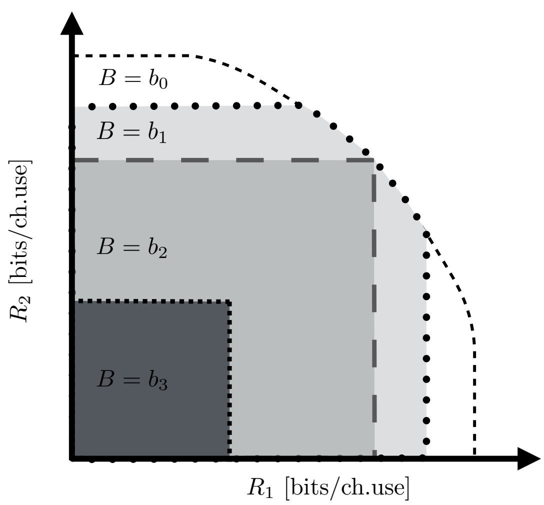
Main Results: 3-D Representation of $\mathcal{E}_0^{\mathrm{FB}}(\mathrm{SNR}_{11}, \mathrm{SNR}_{12}, \mathrm{SNR}_{21}, \mathrm{SNR}_{22})$

$\mathrm{SNR}_{11} = \mathrm{SNR}_{12} = \mathrm{SNR}_{21} = \mathrm{SNR}_{22} = 10$



Main Results: Understanding the Shape of $\mathcal{E}_b^{\mathrm{FB}}(\mathrm{SNR}_{11}, \mathrm{SNR}_{12}, \mathrm{SNR}_{21}, \mathrm{SNR}_{22})$





$$0 \leqslant b_0 \leqslant 1 + \mathrm{SNR}_{21} + \mathrm{SNR}_{22}$$

$$1 + \mathrm{SNR}_{21} + \mathrm{SNR}_{22} \leqslant b_1 < 1 + \mathrm{SNR}_{21} + \mathrm{SNR}_{22} + 2\rho^\star(1,1)\sqrt{\mathrm{SNR}_{21}\mathrm{SNR}_{22}}$$

$$b_2 = 1 + \mathrm{SNR}_{21} + \mathrm{SNR}_{22} + 2\rho^\star(1,1)\sqrt{\mathrm{SNR}_{21}\mathrm{SNR}_{22}}$$

$$1+\mathrm{SNR}_{21}+\mathrm{SNR}_{22}+2\rho^\star(1,1)\sqrt{\mathrm{SNR}_{21}\mathrm{SNR}_{22}} < b_3 \leqslant 1+\mathrm{SNR}_{21}+\mathrm{SNR}_{22}+2\sqrt{\mathrm{SNR}_{21}\mathrm{SNR}_{22}}$$

Proof of Achievability

- At Tx i, with $i \in \{1, 2\}$:
  - β_i : power-splitting coefficient
  - $\beta_i P_i$ to transmit information-carrying (IC) component ([Ozarow'84])
  - $(1 - \beta_i) P_i$ to transmit energy-carrying (EC) component (common randomness)
  - ρ: correlation coefficient between IC-components

- Channel input: $X_{i,t} = \sqrt{(1 - \beta_i) P_i} W_t + U_{i,t}, \quad t \in \{1, \ldots, n\},$
  - EC-symbol: $W_t \sim \mathcal{N}(0, 1)$ is a common randomness known to all terminals.
  - IC-symbols $U_{1,t}$ and $U_{2,t}$ codewords:

- Rx first subtracts common randomness, then performs successive cancellation decoding.

Proof of Converse

- Any $(R_1, R_2, B) \in \mathcal{E}_b(\mathrm{SNR}_{11}, \mathrm{SNR}_{12}, \mathrm{SNR}_{21}, \mathrm{SNR}_{22})$ must satisfy

$$
\begin{aligned}
nR_1 &\leqslant \sum_{t=1}^{n} I(X_{1,t}; Y_{1,t} | X_{2,t}) + \epsilon_1^{(n)}, \\
nR_2 &\leqslant \sum_{t=1}^{n} I(X_{2,t}; Y_{1,t} | X_{1,t}) + \epsilon_2^{(n)}, \\
n(R_1 + R_2) &\leqslant \sum_{t=1}^{n} I(X_{1,t} X_{2,t}; Y_{1,t}) + \epsilon_{12}^{(n)}, \\
B &\leqslant \mathrm{E}\left[B^{(n)}\right] + \delta_n \\
B &\geqslant b
\end{aligned}
$$

where $\frac{\epsilon_1^{(n)}}{n}, \frac{\epsilon_2^{(n)}}{n}, \frac{\epsilon_1^{(n)}}{n}, \delta_n \to 0$ as $n \to \infty$.

- Evaluate these bounds for G-MAC-F with feedback for any $X_{1,t}$ and $X_{2,t}$ with:
  - Mean: $\mu_{i,t} \triangleq \mathrm{E}\left[X_{i,t}\right]$
  - Variance: $\sigma_{i,t}^2 \triangleq \mathrm{Var}(X_{i,t})$
  - Covariance: $\lambda_t \triangleq \mathrm{E}[X_{1,t} X_{2,t}] - \mathrm{E}[X_{1,t}]\,\mathrm{E}[X_{2,t}]$

Main Results: Information-Energy Capacity Region $\mathcal{E}_b(\mathsf{SNR}_{11}, \mathsf{SNR}_{12}, \mathsf{SNR}_{21}, \mathsf{SNR}_{22})$

(Without Feedback)

Theorem: Information-Energy Capacity Region $\mathcal{E}_b(\mathsf{SNR}_{11}, \mathsf{SNR}_{12}, \mathsf{SNR}_{21}, \mathsf{SNR}_{22})$ [B. et al.'15]

$\mathcal{E}_b(\mathsf{SNR}_{11}, \mathsf{SNR}_{12}, \mathsf{SNR}_{21}, \mathsf{SNR}_{22})$ contains all (R_1, R_2, B) that satisfy

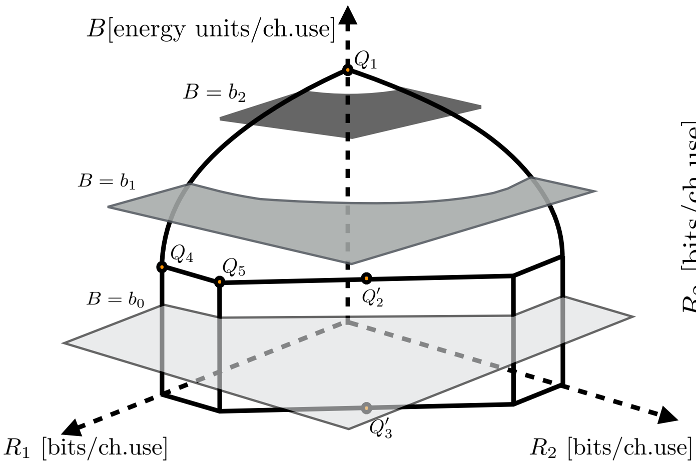
$$\begin{aligned}
0 \leqslant \quad R_1 \quad &\leqslant \frac{1}{2}\log_2\left(1 + \beta_1\,\mathsf{SNR}_{11}\right),\\
0 \leqslant \quad R_2 \quad &\leqslant \frac{1}{2}\log_2\left(1 + \beta_2\,\mathsf{SNR}_{12}\right),\\
0 \leqslant R_1 + R_2 &\leqslant \frac{1}{2}\log_2\left(1 + \beta_1\,\mathsf{SNR}_{11} + \beta_2\,\mathsf{SNR}_{12}\right),\\
b \leqslant \quad B \quad &\leqslant 1 + \mathsf{SNR}_{21} + \mathsf{SNR}_{22} + 2\sqrt{(1-\beta_1)\mathsf{SNR}_{21}(1-\beta_2)\mathsf{SNR}_{22}},
\end{aligned}$$

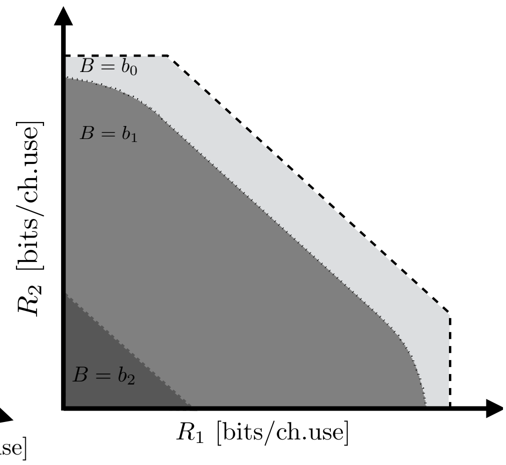
with $(\beta_1, \beta_2) \in [0,1]^2$.

$$\mathcal{E}_b(\mathsf{SNR}_{11}, \mathsf{SNR}_{12}, \mathsf{SNR}_{21}, \mathsf{SNR}_{22}) \subseteq \mathcal{E}_b^{\mathrm{FB}}(\mathsf{SNR}_{11}, \mathsf{SNR}_{12}, \mathsf{SNR}_{21}, \mathsf{SNR}_{22})$$

- Achievability is based on [Cover'75] and [Wyner'74] (Independent IC components)

Understanding the Shape of $\mathcal{E}_b(\mathsf{SNR}_{11}, \mathsf{SNR}_{12}, \mathsf{SNR}_{21}, \mathsf{SNR}_{22})$





$$0 \leqslant b_0 \leqslant 1 + \mathsf{SNR}_{21} + \mathsf{SNR}_{22}$$

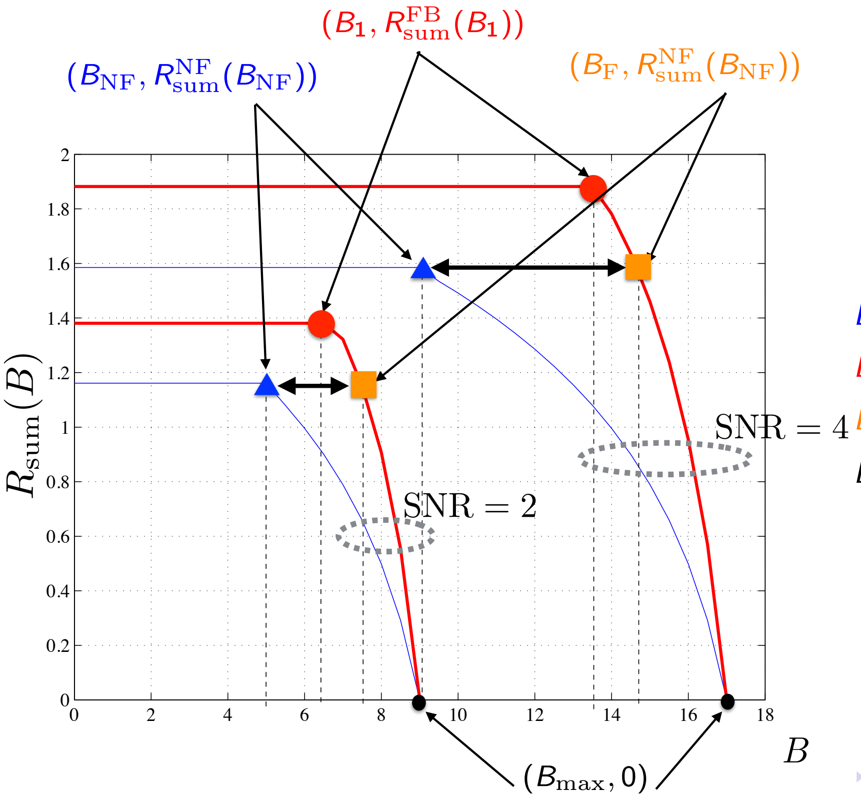
$$1 + \mathsf{SNR}_{21} + \mathsf{SNR}_{22} \leqslant b_1 \leqslant 1 + \mathsf{SNR}_{21} + \mathsf{SNR}_{22} + 2\sqrt{\mathsf{SNR}_{21}\mathsf{SNR}_{22}}\min\left\{\sqrt{\frac{\mathsf{SNR}_{12}}{\mathsf{SNR}_{11}}}, \sqrt{\frac{\mathsf{SNR}_{11}}{\mathsf{SNR}_{12}}}\right\}$$

$$1 + \mathsf{SNR}_{21} + \mathsf{SNR}_{22} + 2\sqrt{\mathsf{SNR}_{21}\mathsf{SNR}_{22}}\min\left\{\sqrt{\frac{\mathsf{SNR}_{12}}{\mathsf{SNR}_{11}}}, \sqrt{\frac{\mathsf{SNR}_{11}}{\mathsf{SNR}_{12}}}\right\} \leqslant b_2 \leqslant 1 + \mathsf{SNR}_{21} + \mathsf{SNR}_{22} + 2\sqrt{\mathsf{SNR}_{21}\mathsf{SNR}_{22}}$$

Does feedback enhance energy transmission?

$R_{sum}^{\mathrm{NF}}(B)$ and $R_{sum}^{\mathrm{FB}}(B)$ as a Function of B

$\mathsf{SNR}_{11} = \mathsf{SNR}_{12} = \mathsf{SNR}_{21} = \mathsf{SNR}_{22} = \mathsf{SNR}$



$B_{\mathrm{NF}} = 1 + 2\mathsf{SNR}$

$B_1 = 1 + 2(1 + \rho^\star)\mathsf{SNR}$

$B_F = ?$

$B_{\mathsf{max}} = 1 + 4\mathsf{SNR}$

Energy Transmission Enhancement With Feedback

- $B_{\mathrm{NF}} \triangleq 1 + \mathsf{SNR}_{21} + \mathsf{SNR}_{22} = \max B$ in G-MAC subject to $R_1 + R_2 = R_{sum}^{\mathrm{NF}}(0)$
- $B_{\mathrm{FB}} \triangleq \max B$ achievable in G-MAC-F when $R_1 + R_2 = R_{sum}^{\mathrm{NF}}(0)$.

Theorem: Maximum Energy Rate B_{F} in G-MAC-F when $R_1 + R_2 = R_{\mathrm{sum}}^{\mathrm{NF}}(0)$

$$B_{\mathrm{F}} = 1 + \mathsf{SNR}_{21} + \mathsf{SNR}_{22} + 2\sqrt{(1-\gamma)\mathsf{SNR}_{21}\mathsf{SNR}_{22}},$$

with $\gamma \in (0,1)$ defined as follows:

$$\gamma \triangleq \frac{\mathsf{SNR}_{11} + \mathsf{SNR}_{12}}{2\mathsf{SNR}_{11}\mathsf{SNR}_{12}} \left[\sqrt{1 + \frac{4\mathsf{SNR}_{11}\mathsf{SNR}_{12}}{\mathsf{SNR}_{11} + \mathsf{SNR}_{12}}} - 1 \right].$$

- Energy enhancement with feedback:

$$\frac{B_{\mathrm{F}}}{B_{\mathrm{NF}}} = 1 + \frac{2\sqrt{(1-\gamma)\mathsf{SNR}_{21}\mathsf{SNR}_{22}}}{1 + \mathsf{SNR}_{21} + \mathsf{SNR}_{22}}.$$

Energy Transmission Enhancement With Feedback

- Let $(\nu_1, \nu_2, \eta_1, \eta_2, \psi_1, \psi_2) \in \mathbb{R}_+^6$ denote
  - $\nu_1 \triangleq \frac{\mathsf{SNR}_{11}}{\mathsf{SNR}_{12}}$ and $\nu_2 \triangleq \frac{\mathsf{SNR}_{12}}{\mathsf{SNR}_{11}}$: asymmetry in channel from TXs to RX
  - $\eta_1 \triangleq \frac{\mathsf{SNR}_{21}}{\mathsf{SNR}_{22}}$ and $\eta_2 \triangleq \frac{\mathsf{SNR}_{22}}{\mathsf{SNR}_{21}}$: asymmetry in channel from TXs to EH
  - $\psi_i \triangleq \frac{\mathsf{SNR}_{2i}}{\mathsf{SNR}_{1i}}$: strength ratio between information and energy channels of TX i

- With these parameters, the energy enhancement ratio can be rewritten as

$$\frac{B_{\mathrm{F}}}{B_{\mathrm{NF}}} = 1 + \frac{2\psi_j \mathsf{SNR}_{1j}\sqrt{\eta_i\left(1 - \left(\frac{1+\nu_i}{2\nu_i\mathsf{SNR}_{1j}}\left(\sqrt{1 + \frac{4\nu_i\mathsf{SNR}_{1j}}{1+\nu_i}} - 1\right)\right)\right)}}{1 + (1+\eta_i)\psi_j\mathsf{SNR}_{1j}}.$$

Energy Transmission Enhancement With Feedback

Corollary: Low SNR Asymptotics

For all $(i,j) \in \{1,2\}^2$ with $i \neq j$, when $\text{SNR}_{1j} \to 0$ while the ratios ν_i, η_i, and ψ_i remain constant, it holds that

$$\lim_{\text{SNR}_{1j} \to 0} \frac{B_{\text{F}}}{B_{\text{NF}}} = 1,$$

and thus feedback does not enhance energy transmission at very low SNR.

Corollary:Maximum Energy Rate Improvement With Feedback

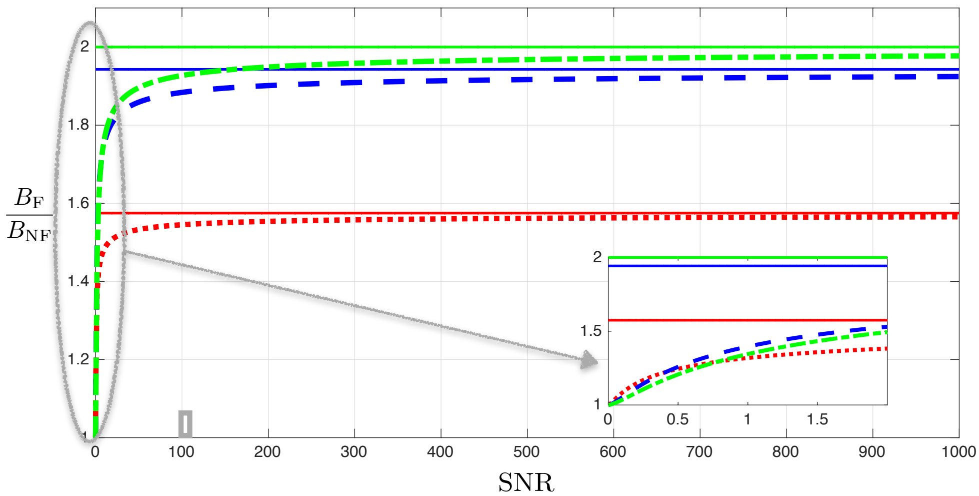
For all $(i,j) \in \{1,2\}^2$ with $i \neq j$, when $\text{SNR}_{1j} \to \infty$ while the ratios ν_i, η_i, and ψ_i remain constant, the maximum energy rate improvement with feedback is given by

$$\lim_{\text{SNR}_{1j} \to \infty} \frac{B_{\text{F}}}{B_{\text{NF}}} = 1 + \frac{2\sqrt{\eta_i}}{1+\eta_i} \leqslant 2.$$

$\Longrightarrow$ Feedback can at most double the energy transmission rate.

Ratio $\frac{B_{\text{F}}}{B_{\text{NF}}}$ and high SNR limit as a function of SNR

Co-located Receiver and Energy Harvester



- High SNR limit (Solid lines).
- $\frac{B_{\text{F}}}{B_{\text{NF}}}$ when $\text{SNR}_{11} = \text{SNR}_{21} = \text{SNR}_1$, $\text{SNR}_{12} = \text{SNR}_{22} = \text{SNR}_2$ and
  - $\text{SNR}_1 = \text{SNR}_2 = \text{SNR}$ (Green–dash-dotted),
  - $\frac{\text{SNR}_1}{2} = \text{SNR}_2 = \text{SNR}$ (Blue–dashed),
  - $\frac{\text{SNR}_1}{10} = \text{SNR}_2 = \text{SNR}$ (Red–dotted), resp.

Conclusion

- Fundamental limits of SEIT in G-MAC-F with minimum energy rate constraint:
  - Information-energy capacity region with and without feedback
  - At very low SNR feedback does not enhance energy transmission
  - Feedback can at most double the energy rate for a fixed information rate.

- Detailed records can be found at

S. Belhadj Amor, S. M. Perlaza, I. Krikidis, and H. V. Poor. "Feedback enhances simultaneous wireless information and energy transmission in multiple access channels", Technical Report, INRIA, No. 8804, Lyon, France, Nov. 2015.

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